2-Period Model and Current Account (1)

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2-Period Model

- 2-Period model enables analyzing how financial transaction, namely, intertemporal transaction affects macroeconomic dynamics.
- This model is fundamental of New Keynesian models which is commonly used to analyze monetary.

Consumption

- Now, we develop a small open economy model which consumes over two periods. Under small open economy models, home country is affected from foreign although home country does not affect foreign country.
- We will show a small open economy's welfare is enhanced by changing the timing of lending and borrowing.

- Households' utility is given by

$$U_1 = u(C_1) + \beta u(C_2) \quad (3.1)$$

where U_1 is households' lifetime utility, C_1 and C_2 denote consumption in periods 1 and 2, respectively, $u(\cdot)$ is an operator of utility function, $\beta \equiv 1/(1+\delta) \in (0,1)$ denotes the subjective discount factor, δ denotes the time preference.

- The higher the time preference, the lower the subjective discount factor. In this case, households do not obtain high utility by consumption in period 2. This implies that the time preference increases if households prefer not future but present consumption.
- Here, the utility function is strictly concave and is an increasing function of consumption.

- Households budget constraint in period 1 is given by:

$$C_1 = Y_1 - B_1^P$$

where Y_1 and B_1^P denote output and households' saving in period 1. This equality shows that the households consume and save when they obtain income.

- Households budget constraint in period 2 is given by:

$$C_2 = Y_2 + (1+r)B_1^P$$

where r denotes real interest rate which is applicable for borrowing and lending in period 1, Y_2 denotes output in period 2. Because of 2-period model, there is not period 3. Households do not save in period 2 and all of output and saving with interest is consumed.

- This economy is a small open economy and is negligible for the rest of the world, namely foreign economy. Thus, as mentioned, home country is affected from foreign although home country does not affect foreign country.
- Further, by assuming complete capital flows, domestic real interest rate corresponds to it in the ROW. That is, the real interest rate is exogenous under this setting.
- For simplicity, that the real interest rate is constant over time is assumed.

- By Combining the budget constraints in periods 1 and 2, we have households' intertemporal budget constraint as follows:

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \qquad (3.2)$$

- Eq.(3.2) shows that the net present value of consumption corresponds to the net present value of output.
- We assume perfect forsight.

- Households maximize Eq.(3.1) subject to Eq.(3.2). This maximization problem is given by:

$$\max_{C_1, C_2} u(C_1) + \beta u(C_2)$$

- This problem is one of typical optimization problem with constraints.
- While this problem can be solved through plugging Eq.(3.2) into this problem, we solve this problem through Lagrange multiplyer method.

- The Lagrangean is given by:

$$L \equiv u(C_1) + \beta u(C_2) + \lambda \left(Y_1 + \frac{Y_2}{1+r} - C_1 - \frac{C_2}{1+r} \right)$$

where λ is Lagrange multiplier.

- The first order necessary conditions are given by:

$$u'(C_1) - \lambda = 0$$

$$\beta u'(C_2) - \frac{\lambda}{1+r} = 0$$

- By combining these FONCs, we have:

$$u'(C_1) = (1+r)\beta u'(C_2) \qquad (3.3)$$

- Eq.(3.3) is so called intertemporal Euler equation.
- This Euler equation shows that households' utility is no longer enhanced even if households choose any consumption schedule under utility maximization phenomenon.

- Now, let assume that households decrease one unit of consumption in period 1. While households' lifetime utility U_1 decreases as $u'(C_1)$, consumption in period 2 increases as 1+r. This increases lifetime utility as (1+r)$u'(C_2)$. This increase cancel that decrease.
- Thus, households' utility is no longer enhanced, even if another consumption schedule is chosen.

- Eq.(3.3) can be rewritten as:

$$\beta\left(\frac{u'(C_2)}{u'(C_1)}\right)=\frac{1}{1+r} \qquad (3.4)$$

- The LHS in Eq.(3.4) is the marginal rate of substitution between consumption in periods 1 and 2, while the RHS in Eq.(3.4) is the price of consumption in period 2 in terms of consumption in period 1.

- Eq.(3.3) shows that the representative household's optimal consumption schedule depends on both subjective discount factor and the marginal rate of substitution on consumption.
- If the subjective discount factor is smaller than the marginal rate of substitution, that is, $\beta<1/(1+r)$, $u'(C_1)<u'(C_2)$ is applied.

- Now, $u''<0$, that is, the marginal utility of consumption gradually decreases. Thus, there is enough consumption if the marginal utility of consumption is small.
- Then, $u'(C_1)<u'(C_2)$ is applied, the consumption in period 1 is larger than it in period 2.

- If $\beta<1/(1+r)$, that is, $r<\delta$ which implies that the real interest rate is smaller than the rate of time preference, households can not obtain enough interest in period 2 as a reward of enduring consumption in period 1.
- In that case, households shall increase consumption in period 1.
- If the subjective discount factor is larger than the marginal rate of substitution, C_1<C_2 is applied on the same score.

- The subjective discount factor is equal to the marginal rate of substitution, that is, $\beta=1/(1+r)$ is applied, Eq.(3.3) boilds down to $u'(C_1)=u'(C_2)$.
- In that case, $C_1=C_2$ is also applied and households' consumption in periods 1 and 2 are same. That is, consumption is constant over time.

- We focus on this special case hereafter.
- Plugging $C_1 = C_2$ into Eq.(3.2) yields:

$$\bar{C} = \frac{(1+r)Y_1 + Y_2}{2+r} \qquad (3.5)$$

where $\bar{C} \equiv C_1 = C_2$. Because of constant consumption, we adopt this definition.

International Indebtedness

- Current account is a change of net external assets. If the capital accumulation is negligible, the current account in period 1 corresponds to saving.
- If the current account is in black, a country lend out. If it is in red, a country borrows.
- Here, we define that export of services includes a value of home capital's activity in foreign, namely, interest income stemming from holding net external assets.

- Similarly, we define that import of services includes a value of foreign capital's activity in home, namely, interest expense.
- The fact that current account corresponds to an increase in net external assets implies that (positive) net export of goods and services accompanies with an increase in external assets.
- The reason is that sales on goods and services for foreign exceeds buying those from foreign.

- Similarly, negative net export of goods and services, namely, the excess of imports over exports, accompanies with a decrease in external assets.
- Because home receives payment from foreign when home exports goods and services, (positive) net export is synonym of net import of capital. In that case, capital account is in red.
- The sum of current account and capital account is definitely zero, if foreign currency reserve is negligible. Because of this, net increase in external assets coincides with capital account deficit.

- Now we show the concept of current account. Let define the (balance of) net external assets in the end of period t as B_{t+1}. Because current account corresponds to a change in net external assets and to net exports of goods and services (including interest income), we have following:

$$\begin{aligned} CA_t &= B_{t+1} - B_t \\ &= Y_t + r_t B_t - C_t \qquad (3.0) \end{aligned}$$

where CA_t denotes the current account in period t.

- In Eq.(3.6), $r_t B_t$ is interest income by holding net external assets. In addition, Eq.(3.0) shows that current account corresponds to the difference between income (including interest income) and consumption.
- This fact is obvious because Y_t is GDP, $r_t B_t$ is interest income and $Y_t + r_t B_t$ is Gross National Product, another concept of national income.
- Note that the second equality in Eq.(3.0) is just applicable if both the capital accumulation and the government expenditure are negligible.

- Under an open economy where is international indebtedness, home consumption does not necessarily correspond to home output.
- Here, we assume that all of loans are redeemed with interest payment absolutely and the budget constraint Eq.(3.2) is still applied.
- Here, consumption is constant over time while output is not necessarily so.
- Now, let imagine that the home in the model is country with current account deficit, such as US.

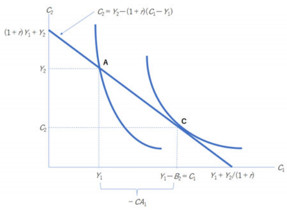
- In that case, home borrows $\bar{C} - Y_1$ from foreign and refund $(1+r)(\bar{C} - Y_1)$ in period 2. Thus, consumption in period 2 is equal to the difference between output and borrowing with interest payment and is given by:
$$C_2 = Y_2 - (1+r)(C_1 - Y_1) \qquad (3.6)$$
- Because Eq.(3.6) is the budget constraint Eq.(3.2) it self, budget constraint is sufficed even if $Y_1 < Y_2$, that is, borrowing in period 1 and refunding in period 2.
- This is applied in the opposite case, $Y_1 > Y_2$. Thus, output not need to be consistent over time.

Consumption and Current Account

- Home consumption does not necessarily correspond to home output if there is international indebtedness, as mentioned. Now, we discuss on how international indebtedness changes households' consumption schedule.
- To discuss on, we have to derive indifferent curve and we derive it in 2-period model.
- In 2-periods model, we have indifferent curve on consumption in periods 1 and 2, respectively.

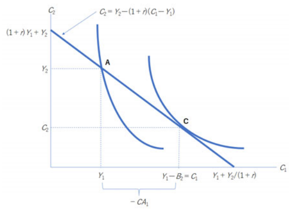
- The FONC for households for maximizing Eq.(3.1) is given by:
$$u'(C_1)dC_1 + \beta u'(C_2)dC_2 = 0$$
- By dividing dC_1 both sides, we get:
$$\frac{dC_2}{dC_1} = -\frac{u'(C_1)}{\beta u'(C_2)} \qquad (3.8)$$
- In the LHS in Eq.(3.8) is the marginal rate of intertemporal substitution on consumption. In the RHS shows that the slope is negative. Thus, Eq.(3.8) is obviously the slope of indifferent curve.

Fig. 3-1: Intertemporal Consumption and Current Account



- Fig. 3 shows households' budget constraint Eq.(3.2) and indifferent curve Eq.(3.8). Here, Eq.(3.2) is arranged as Eq. (3.6).
- The optimal combination of consumption in periods 1 and 2 is given by *C* where the indifferent curve adjoins the budget constraint.

Fig. 3-1: Intertemporal Consumption and Current Account



- In period 1, consumption exceeds output and the current account is in red.
- In period 2, on contrary, output exceeds consumption and the current account is in black.

Government

- While we omitted government, we introduce it here and show the role of government. If the government levies tax on households' income, households' budget constraint is given by:

$$C_1 + \frac{C_2}{1+r} = Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}$$

in stead of Eq.(3.2). Here, T_1 and T_2 denote the tax in periods 1 and 2, respectively. Then, Y_1—T_1 and Y_2—T_2 are disposable income in periods 1 and 2, respectively.

- Similar to households' budget constraint, if the government's budget constraints are given by $G_1 = T_1 + B_1^G$ and $G_2 = T_2 - (1+r)B_1^G$, the former equality can be rewritten as:

$$C_1 + \frac{C_2}{1+r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+r} \qquad (3.9)$$

where G_1 and G_2 denote the government expenditure in periods 1 and 2, respectively and B_1^G denotes the government debt.

- By combining the government's budget constraint in each period, we have the government's intertemporal budget constraint as follows

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

- Then, the definition of the current account Eq.(3.0) should be written as:

$$\begin{aligned} CA_t &= B_{t+1} - B_t \\ &= Y_t + rB_t - C_t - G_t \end{aligned} \qquad (3.10)$$

- Different from Eq.(3.0), Eq.(3.10) shows that the current account corresponds to not the difference between national income and consumption but the difference between national income and the sum of consumption and the government expenditure.
- Because government expenditure is one of domestic expenditure, namely, absorption, the current account must correspond to the difference between national income and the sum of consumption and the government expenditure.

- Because households cannot control the government expenditure, Euler equation Eq.(3.3) is still applied.
- Now, we assume that $\beta = 1/(1+r)$.
- By assuming output is constant over time, namely, $Y_1 = Y_2 \equiv \overline{Y}$, we consider how a change in government expenditure affects consumption.
- If there is not the government expenditure, Eq.(3.5) is applicable and consumption is constant over time. In that case, if the output is constant over time, $\overline{Y} = \overline{C}$ is applied and there is balanced current account.

- However, if G_1>T_1, that is, the government expenditure exceeds tax revenue in period 1 and there is fiscal deficit, and G_2=0, that is, there is no government expenditure in period 2, how consumption and the current account change?

- Plugging government's budget constraint into Eq.(3.5) yields:

$$\bar{C} = \frac{(1+r)(\bar{Y}-G_1)+\bar{Y}}{2+r}$$
$$= \bar{Y} - \frac{(1+r)G_1}{2+r}$$

This equality implies that the fiscal deficit in period 1 decreases consumption although this decrease is smaller than an increase G_1 in magnitude.

- The reason is that an increase in government expenditure is temporally and there is no government expenditure in period 2.

- Plugging this into Eq.(3.10) yields:

$$CA_1 = \bar{Y} - \bar{C} - G_1$$
$$= -\frac{G_1}{2+r} < 0$$

which implies that the current account in period 1 is red.

- As shown in Eq.(3.3), households seek to make consumption constant over time and part of disposable income in period 2 which is higher than in in period 1 is applied to a part of disposable income in period 1.
- That is, home borrows in period 1. Then, the current account is in red in period 1 while it is in black in period 2.

Investment

- In an economy where is no investment, current account corresponds to domestic saving. However, generally speaking, current account corresponds to the difference between saving and investment.
- Historically, an economy without enough saving depends on borrowing from foreign to invest and investment is more volatile than consumption.
- Considering these facts, investment is important to discuss on fluctuation in current account.

- In fact, the current account deficit in developing countries is results from investment depending on borrowing from developed countries and the US under IT boom (1999--2000) where investment was prosperous was visited by expanding current account deficit.

- Now we consider optimization problem faced by firms investing. Investment is used to production. Firm's production function is given by:

$$Y_t = F(K_t) \quad (3.11)$$

where K_t denotes capital. We adopt familiar assumption that the marginal product of capital is diminished. That is, $F'(\cdot) > 0$ and $F''(\cdot) < 0$.

- Capital is formed by accumulating investment. For simplicity, we assume that capital wastage is negligible.

- Process of capital accumulation is given by:

$$K_{t+1} = K_t + I_t \quad (3.12)$$

where I_t denotes investment.
- We assume that capital goods, which is used to investment is same as goods to be consumed. At glance, this assumption is extreme. However, this assumption does not affect the main result.

- Although households maximize their utility, firms maximize their profit. Firms' optimization problem is given by:

$$\max_{I_1, I_2} \left[F(K_1) - I_1 \right] + \frac{F(K_2) - I_2}{1+r} \quad (3.13)$$

This shows that firms maximize their profit by choosing investment in periods 1 and 2.
- In Eq.(3.13), I_1 denotes the investment in period 1 and K_1 and K_2 denote the capital in periods 1 and 2. Thus, the first term is profit in period 1 and the second term is net present value of profit in period 2.

- As mentioned, this model comes to end in the end of period 2. Thus, any capital does not remain in the end of period 2. That is, K_3=0 is applied. Because of this, the investment in period 2 is given by:

$$\begin{aligned} I_2 &= K_3 - K_2 \\ &= -K_2 \end{aligned}$$

- Paying attention to this and Eq.(3.12), the FONC for firms is given by:

$$-1 + \frac{1}{1+r}\left[1 + \frac{\partial F(K_2)}{\partial K_2}\right] = 0$$

Now, $\frac{\partial K_2}{\partial I_1} = 1$ is applicable.

- Rearranging this FONC yields:

$$F'(K_2) = r$$

- This equality implies that firms maximizing their profit invest to equalize the marginal product of capital to interest rate.
- Further, the lower the interest rate, the higher the investment and vice versa because of diminished marginal product of capital.
- Thus, the investment is a decreasing function of the interest rate.